Q1a

a) Rewrite the numbers inside the functions as powers of their bases:

$$4\log_3 3^6 + 3\log_2 2^{12} - 3\log 10^2 + 6$$

Simplify,

$$= 4(6) + 3(12) - 3(2) + 6$$

$$= \boxed{60}$$

Q1b

b) Rewrite the coefficients as powers

$$\ln\sqrt{196} + \ln\sqrt[3]{125} + \ln\sqrt[4]{81} + \ln\sqrt[5]{32}$$

$$= \ln 14 + \ln 5 + \ln 3 + \ln 2$$

Combine the terms using log law of addition

$$= \ln(14 \times 5 \times 3 \times 2)$$

$$= \boxed{\ln 420}$$

Q2

Spot the hidden quadratic!

$2(5)(5^x)^2 - 41(5^x) + 21 = 0$

Let $y = 5^x$

$10y^2 - 41y + 21 = 0$

$(5y-3)(2y-7) = 0$

$y = \frac{7}{2}, \frac{3}{5}$

$5^x = \frac{7}{2}, \quad 5^x = \frac{3}{5}$

$\boxed{x = \log_5 \frac{7}{2} \qquad x = \log_5 \frac{3}{5}}$

Q3a

a) $e^{3x^2-1} = \frac{12}{8} = \frac{3}{2}$

$\ln e^{3x^2-1} = \ln \frac{3}{2}$

$3x^2 - 1 = \ln \frac{3}{2}$

$x = \sqrt{\frac{\ln \frac{3}{2} + 1}{3}}$

$= \boxed{0.684, -0.684}$ (3sf)

Q3b

b) $e^{3x} - 42 = 12(e^x)^2 - 14e^x$

Spot the hidden cubic!

$(e^x)^3 - 42 = 12(e^x)^2 - 14(e^x)$

$(e^x)^3 - 12(e^x)^2 + 14e^x - 42 = 0$

Let $y = e^x$

$y^3 - 12y^2 + 14y - 42 = 0$

$y = 11.0$

$e^x = 11.0$

$x = \ln 11.0 = \boxed{2.41}$ (3sf)

Q4

Rewrite coefficients as powers

LHS: $\log_3 x^2 + \log_3 (x^2-1) - \log_3 (x+1)^2$

difference of two squares

$= \log_3 \frac{x^2(x^2-1)}{(x+1)^2} = \log_3 \frac{x^2\cancel{(x+1)}(x-1)}{(x+1)^{\cancel{2}\,1}}$

$= \log_3 \frac{x^2(x-1)}{x+1} = \text{RHS}$

$\boxed{\therefore \text{LHS} = \text{RHS}}$

Q5

Rewrite coefficients as powers

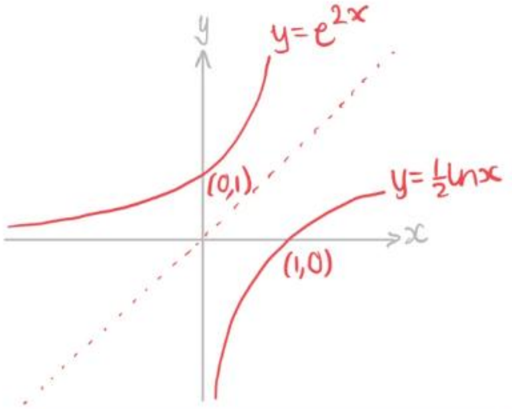
$\log_p(x+1)^2 + \log_p(x-1)^3 - \log_p(x^2-1)$

Combine using log laws of addition & subtraction

$\log_p \frac{(x+1)^2(x-1)^3}{x^2-1}$ difference of 2 squares!

$\log_p \frac{(x+1)^{\cancel{2}\,1}(x-1)^{\cancel{3}\,2}}{\cancel{(x+1)}\cancel{(x-1)}} = \boxed{\log_p (x+1)(x-1)^2}$

Q6



$\boxed{\text{y = x is the line of reflection}}$

(the functions are the inverse of one another, this can be shown mathematically too!)

$y=e^{2x}$

$\ln y = 2x$

$x = \frac{1}{2}\ln y$

reflect along $y=x$

$y=\frac{1}{2}\ln x$

asymptote for $y=e^{2x}$ is $y=0$
asymptote for $y=\frac{1}{2}\ln x$ is $x=0$

Q7

$4 - \ln 2^4$

$= 4 - 4\ln 2$

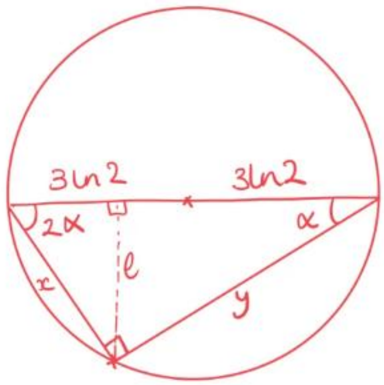
$= 4(1 - \ln 2)$

by writing 1 as $\ln e$

$= 4(\ln e - \ln 2)$

$= \boxed{4\ln\left(\frac{e}{2}\right)}$

Q8



circle theorum:

angles subtended by diameter are 90°.

$180° = 90° + 3\alpha$

$\alpha = 30°$

pythagorus' theorum

$(6\ln 2)^2 = x^2 + y^2$ ①

trig!

$\sin 2\alpha = \frac{l}{x}$ $\quad \sin\alpha = \frac{l}{y}$

eliminate l

$x\sin 2\alpha = y\sin\alpha$

② $y = \frac{x\sin 2(30)}{\sin 30} = \sqrt{3}\,x$

3 sides: $\boxed{\begin{matrix}6\ln 2\\ 3\ln 2\\ 3\sqrt{3}\ln 2\end{matrix}}$

solve simultaneous eqns ① & ②

sub $y = \sqrt{3}x$ into ①

$(6\ln 2)^2 = x^2 + (\sqrt{3}x)^2 = 4x^2$

$x = \frac{6\ln 2}{2} = 3\ln 2$

sub $x = 3\ln 2$ into ②

$y = \sqrt{3}(3\ln 2) = 3\sqrt{3}\ln 2$

Q9

$$\log_x (x+1)^3 = 3$$

$$x^3 = (x+1)^3$$

$$= (x+1)(x^2 + 2x + 1)$$

$$\cancel{x^3} = \cancel{x^3} + 3x^2 + 3x + 1$$

$$0 = 3x^2 + 3x + 1$$

discriminant

$$b^2 - 4ac = 3^2 - 4(3)(1) = -3$$

$$-3 < 0$$

$$\boxed{\text{NO REAL SOLUTIONS since discriminant} < 0}$$

Q10

LHS	RHS
$8 = 2^3 = (\sqrt{4})^3 = (4^{\frac{1}{2}})^3 = 4^{\frac{3}{2}}$	$27 = 3^3 = (\sqrt{9})^3 = (9^{\frac{1}{2}})^3 = 9^{\frac{3}{2}}$
$\log_4 4^{\frac{3}{2}}$	$\log_9 9^{\frac{3}{2}}$
$= \frac{3}{2}\underbrace{\log_4 4}_{1}$	$= \frac{3}{2}\underbrace{\log_9 9}_{1}$
$= \frac{3}{2}$	$= \frac{3}{2}$

$$\boxed{\therefore \text{LHS} = \text{RHS}}$$